BIMETALLIC STRIP

INPUT: (a material is on top and b material is on bottom)

$t_a := 2.0$ $t_b := 2.0$ (Thicknesses of a and b, in) $w := 1$ (width, in) $le := 1$ (Length of strip, in)

$E_a := 10000000$ $E_b := 30000000$ (Young's modulus of a and b, psi)

$\alpha_a := 12\cdot 10^{-6}$ $\alpha_b := 6\cdot 10^{-6}$ (Coefficients of thermal expansion (CTE) for a and b, in/in/deg F)

$M := -10000$ (Bending moment, in-lb) $\Delta T := 100$ (Temperature difference, deg F)

Roark and Young Equations, p112:

$$K_1 := 4 + 6\cdot\frac{t_a}{t_b} + 4\cdot\left(\frac{t_a}{t_b}\right)^2 + \frac{E_a}{E_b}\cdot\left(\frac{t_a}{t_b}\right)^3 + \frac{E_b}{E_a}\cdot\frac{t_b}{t_a} \qquad K_1 = 17.333$$

$$EI := \frac{w\cdot t_b^3\cdot t_a\cdot E_b\cdot E_a}{12\cdot\left(t_a\cdot E_a + t_b\cdot E_b\right)}\cdot K_1 \qquad EI = 8.667\cdot 10^7$$ (Bending stiffness, lb-in^2/in)

Mechanical Bending Stresses:

$$\sigma_{aupperm} := \frac{-6\cdot M}{w\cdot t_b^2\cdot K_1}\cdot\left(2 + \frac{t_b}{t_a} + \frac{E_a\cdot t_a}{E_b\cdot t_b}\right) \qquad \sigma_{aupperm} = 2.885\cdot 10^3$$ (Stress top of a, psi)

$$\sigma_{blowerm} := \frac{6\cdot M}{w\cdot t_b^2\cdot K_1}\cdot\left(2 + \frac{t_a}{t_b} + \frac{E_b\cdot t_b}{E_a\cdot t_a}\right) \qquad \sigma_{blowerm} = -5.192\cdot 10^3$$ (Stress bottom of b, psi)

Thermal Stresses:

$$\sigma_{auppert} := \frac{-\left(\alpha_b - \alpha_a\right)\cdot\Delta T\cdot E_a}{K_1}\cdot\left[3\cdot\frac{t_a}{t_b} + 2\cdot\left(\frac{t_a}{t_b}\right)^2 - \frac{E_b\cdot t_b}{E_a\cdot t_a}\right] \qquad \sigma_{auppert} = 692.308$$ (Stress top of a, psi)

$$\sigma_{blowert} := \frac{\left(\alpha_b - \alpha_a\right)\cdot\Delta T\cdot E_b}{K_1}\cdot\left[3\cdot\frac{t_a}{t_b} + 2 - \frac{E_a}{E_b}\cdot\left(\frac{t_a}{t_b}\right)^3\right] \qquad \sigma_{blowert} = -4.846\cdot 10^3$$ (Stress bottom of b, psi)

Total Stresses:

$\sigma_{aupper} := \sigma_{aupperm} + \sigma_{auppert}$ $\sigma_{aupper} = 3.5769\cdot 10^3$ (Stress top of a, psi)

$\sigma_{blower} := \sigma_{blowerm} + \sigma_{blowert}$ $\sigma_{blower} = -1.0038\cdot 10^4$ (Stress bottom of b, psi)

Timoshenko, Journal of the Optical Society of America, v11, 1925

Thermal Stress Calculations:

$h := t_a + t_b \quad h = 4.000 \qquad I_a := \frac{t_a^3}{12} \quad I_a = 0.667 \qquad I_b := \frac{t_b^3}{12} \quad I_b = 0.667$

$$\kappa := \frac{(\alpha_b - \alpha_a)\cdot \Delta T}{\frac{h}{2} + \frac{2\cdot(E_a \cdot I_a + E_b \cdot I_b)}{h}\cdot\left(\frac{1}{E_a \cdot t_a} + \frac{1}{E_b \cdot t_b}\right)} \quad \kappa = -2.077\cdot 10^{-4}$$ (Curvature, 1/in)

$\rho := \frac{1}{\kappa} \quad \rho = -4.815\cdot 10^3$ (Radius of curvature, in)

$P := \frac{2\cdot(E_a \cdot I_a + E_b \cdot I_b)}{h\cdot\rho} \quad P = -2.769\cdot 10^3$ (Interface force, lb)

$M_a := \frac{E_a \cdot I_a}{\rho} \quad M_a = -1.385\cdot 10^3 \quad M_b := \frac{E_b \cdot I_b}{\rho} \quad M_b = -4.154\cdot 10^3$ (Bending moments in a and b, in-lb)

$\sigma_{amax} := \frac{P}{t_a} + \frac{t_a \cdot E_a}{2\cdot\rho} \quad \sigma_{amax} = -3.462\cdot 10^3$ (At interface of a) (Stress, psi)

$\sigma_{amax} := \frac{1}{\rho}\cdot\left[\left(\frac{2}{h\cdot t_a}\right)\cdot(E_a \cdot I_a + E_b \cdot I_b) + \frac{t_a \cdot E_a}{2}\right] \quad \sigma_{amax} = -3.462\cdot 10^3$ (At interface of a) (Stress, psi)

$\sigma_{amin} := \frac{P}{t_a} - \frac{t_a \cdot E_a}{2\cdot\rho} \quad \sigma_{amin} = 692.308$ (At top surface of a) (Stress, psi)

$\sigma_{bmax} := \frac{-P}{t_b} - \frac{t_b \cdot E_b}{2\cdot\rho} \quad \sigma_{bmax} = 7.615\cdot 10^3$ (At interface of b) (Stress, psi)

$\sigma_{bmin} := \frac{-P}{t_b} + \frac{t_b \cdot E_b}{2\cdot\rho} \quad \sigma_{bmin} = -4.846\cdot 10^3$ (At bottom surface of b) (Stress, psi)

$\delta := \frac{le^2}{8\cdot\rho} \quad \delta = -2.596\cdot 10^{-5}$ (Deflection, in)

Mechanical Stress Calculations:

$n := \frac{E_a}{E_b}$ $n = 0.333$ $m := \frac{t_a}{t_b}$ $m = 1.000$

$na := \frac{E_a \cdot t_a \cdot \left(\frac{t_a}{2}\right) + E_b \cdot t_b \cdot \left(\frac{t_b}{2} + t_a\right)}{E_a \cdot t_a + E_b \cdot t_b}$ $na = 2.500$ (neutral axis from top of laminate, in)

$I := \frac{(t_a + t_b)^3}{3 \cdot n} + \left(1 - \frac{1}{n}\right) \cdot \frac{t_a^3}{3} - \left(t_a + \frac{t_b}{n}\right) \cdot na^2$ $I = 8.667$ (Moment of inertia per Timoshenko for unit width)

$EI := \frac{t_a^3 \cdot E_a}{12} + \frac{t_b^3 \cdot E_b}{12} + t_a \cdot E_a \cdot \left(na - \frac{t_a}{2}\right)^2 + t_b \cdot E_b \cdot \left(t_a + \frac{t_b}{2} - na\right)^2$ $EI = 8.667 \cdot 10^7$

(Bending stiffness, lb-in^2/in)

$na := \frac{E_a \cdot t_a \cdot \left(t_b + \frac{t_a}{2}\right) + E_b \cdot t_b \cdot \left(\frac{t_b}{2}\right)}{E_a \cdot t_a + E_b \cdot t_b}$ $na = 1.500$ (neutral axis from bottom of laminate, in)

$I := \frac{1}{12} \cdot \frac{E_b}{E_a} t_b^3 \cdot w + \frac{1}{12} \cdot \frac{E_a}{E_a} \cdot t_a^3 \cdot w + \frac{E_b}{E_a} \cdot w \cdot t_b \cdot \left(na - \frac{t_b}{2}\right)^2 + \frac{E_a}{E_a} \cdot w \cdot t_a \cdot \left(t_b + \frac{t_a}{2} - na\right)^2$ $I = 8.667$

(Moment of inertia, lb-in^2)

Stresses in top layer a, psi:

(Distance from bottom of laminate, in) $c := t_a + t_b, t_a + t_b - .25 .. t_a$

$\sigma(c) := -\frac{M \cdot (c - na)}{I}$

$\sigma(c) =$	$c =$
$2.885 \cdot 10^3$	4.000
$2.596 \cdot 10^3$	3.750
$2.308 \cdot 10^3$	3.500
$2.019 \cdot 10^3$	3.250
$1.731 \cdot 10^3$	3.000
$1.442 \cdot 10^3$	2.750
$1.154 \cdot 10^3$	2.500
865.385	2.250
576.923	2.000

Stresses in bottom layer b, psi: $c := t_b, t_b - .25 .. 0$

(Distance from bottom of laminate, in)

$\sigma(c) := -\frac{E_b}{E_a} \cdot M \cdot \frac{(c - na)}{I}$

$\sigma(c) =$	$c =$
$1.731 \cdot 10^3$	2.000
865.385	1.750
0.000	1.500
-865.385	1.250
$-1.731 \cdot 10^3$	1.000
$-2.596 \cdot 10^3$	0.750
$-3.462 \cdot 10^3$	0.500
$-4.327 \cdot 10^3$	0.250
$-5.192 \cdot 10^3$	0.000

$E_{eff} := \frac{E_a \cdot t_a + E_b \cdot t_b}{t_a + t_b}$ $E_{eff} = 2.000 \cdot 10^7$ (Effective modulus, psi)

$CTE_{eff} := \frac{E_a \cdot t_a \cdot \alpha_a + E_b \cdot t_b \cdot \alpha_b}{E_a \cdot t_a + E_b \cdot t_b}$ $CTE_{eff} = 7.500 \cdot 10^{-6}$ (Effective CTE, in/in/deg F)

Laminate Thermal Load Analysis using Classical ABD Formulation:

$h_t := \frac{h}{2}$ $h_t = 2.000$ $h_i := h_t - t_a$ $h_i = 0.000$ $h_b := \frac{-h}{2}$ $h_b = -2.000$

$A11 := E_a \cdot t_a + E_b \cdot t_b$ $A11 = 8.000 \cdot 10^7$ $B11 := \frac{E_a}{2} \cdot \left(h_t^2 - h_i^2\right) + \frac{E_b}{2} \cdot \left(h_i^2 - h_b^2\right)$ $B11 = -4.000 \cdot 10^7$

$Nxt := E_a \cdot \Delta T \cdot t_a \cdot \alpha_a + E_b \cdot \Delta T \cdot t_b \cdot c Nxt = 6.000 \cdot 10^4$

$D11 := \frac{1}{3} \cdot \left[\left(h_t^3 - h_i^3\right) \cdot E_a + \left(h_i^3 - h_b^3\right) \cdot E_b\right]$ $D11 = 1.067 \cdot 10^8$

$Mxt := \frac{1}{2} \cdot \left[\left(h_t^2 - h_i^2\right) \cdot \alpha_a \cdot \Delta T \cdot E_a + \left(h_i^2 - h_b^2\right) \cdot \alpha_b \cdot \Delta T \cdot E_b\right]$ $Mxt = -1.200 \cdot 10^4$ (Thermal moment, in lb/in)

$a44 := \frac{A11}{A11 \cdot D11 - B11^2}$ $a44 = 1.154 \cdot 10^{-8}$

$a11 := \frac{D11}{A11 \cdot D11 - B11^2}$ $a11 = 1.538 \cdot 10^{-8}$ $a14 := \frac{-B11}{A11 \cdot D11 - B11^2}$ $a14 = 5.769 \cdot 10^{-9}$

$\varepsilon_x := a11 \cdot Nxt + a14 \cdot M \rangle \varepsilon_x = 8.538 \cdot 10^{-4}$

(Mid-plane strains and curatures)

$\kappa_x := a14 \cdot Nxt + a44 \cdot Mxt$ $\kappa_x = 2.077 \cdot 10^{-4}$

$h := h_i, h_i + .2 .. h_t$ (Zero is at mid plane)

$\sigma_a(h) := \left(\varepsilon_x + \kappa_x \cdot h - \alpha_a \cdot \Delta T\right) \cdot E_a$

$\sigma_a(h) =$	$h =$
$-3.462 \cdot 10^3$	0.000
$-3.046 \cdot 10^3$	0.200
$-2.631 \cdot 10^3$	0.400
$-2.215 \cdot 10^3$	0.600
$-1.800 \cdot 10^3$	0.800
$-1.385 \cdot 10^3$	1.000
-969.231	1.200
-553.846	1.400
-138.462	1.600
276.923	1.800
692.308	2.000

(Stress in top material a, psi)

(Stress in bottom material b, psi)

$\sigma_b(h) := \left(\varepsilon_x + \kappa_x \cdot h - \alpha_b \cdot \Delta T\right) \cdot E_b$

$h := h_i, h_i - .2 .. h_b$

(Zero is at mid plane)

$\sigma_b(h) =$	$h =$
$7.615 \cdot 10^3$	0.000
$6.369 \cdot 10^3$	-0.200
$5.123 \cdot 10^3$	-0.400
$3.877 \cdot 10^3$	-0.600
$2.631 \cdot 10^3$	-0.800
$1.385 \cdot 10^3$	-1.000
138.462	-1.200
$-1.108 \cdot 10^3$	-1.400
$-2.354 \cdot 10^3$	-1.600
$-3.600 \cdot 10^3$	-1.800
$-4.846 \cdot 10^3$	-2.000

Laminate Mechanical Load Analysis using Advanced ABD Formulation (Tsai and Hahn, Introduction to Composite Materials):

$h := t_a + t_b$ $h = 4.000$ $A11 = 8.000 \cdot 10^7$ $na = 1.500$ (From bottom of laminate, in)

$D11 := D11 - 2 \cdot \left(na - \frac{h}{2}\right) \cdot B11 + \left(na - \frac{h}{2}\right)^2 \cdot A11$ $D11 = 8.667 \cdot 10^7$ $a44 := \frac{1}{D11}$ $a44 = 1.154 \cdot 10^{-8}$

$B11 := B11 - \left(na - \frac{h}{2}\right) \cdot A11$ $B11 = 0.000$

$\kappa_x := a44 \cdot M$ $\kappa_x = -1.154 \cdot 10^{-4}$

$h := \frac{h}{2} - na, \frac{h}{2} - na + .2 .. h - na$ (Zero is at neutral axis)

$\sigma_a(h) := -(\kappa_x \cdot h) \cdot E_a$ (Stress in top material a, psi)

$\sigma_a(h) =$	$h =$
576.923	0.500
807.692	0.700
$1.038 \cdot 10^3$	0.900
$1.269 \cdot 10^3$	1.100
$1.500 \cdot 10^3$	1.300
$1.731 \cdot 10^3$	1.500
$1.962 \cdot 10^3$	1.700
$2.192 \cdot 10^3$	1.900
$2.423 \cdot 10^3$	2.100
$2.654 \cdot 10^3$	2.300
$2.885 \cdot 10^3$	2.500

$h := t_a + t_b$

$h := \frac{h}{2} - na, \frac{h}{2} - na - .2 .. -na$ (Stress in bottom material b, psi)

$\sigma_b(h) := -(\kappa_x \cdot h) \cdot E_b$

$\sigma_b(h) =$	$h =$
$1.731 \cdot 10^3$	0.500
$1.038 \cdot 10^3$	0.300
346.154	0.100
-346.154	-0.100
$-1.038 \cdot 10^3$	-0.300
$-1.731 \cdot 10^3$	-0.500
$-2.423 \cdot 10^3$	-0.700
$-3.115 \cdot 10^3$	-0.900
$-3.808 \cdot 10^3$	-1.100
$-4.500 \cdot 10^3$	-1.300
$-5.192 \cdot 10^3$	-1.500

(Zero is at neutral axis)